Capacitive Mains Inputs; choosing capacitor and resistor combinations for 120 V a.c. 240 V a.c and 50 and 60Hz

R.P. clark

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Abstract

Calculations to work out capacitance values to drive an opto-coupler to detect mains voltage for 50 to 60 Hz.

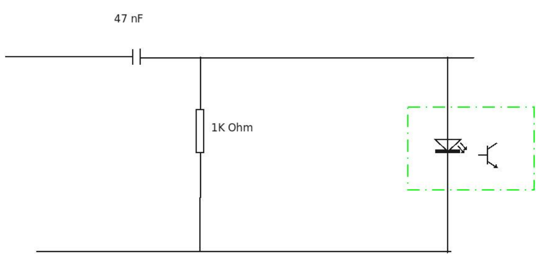

Figure 1: Opto-coupled mains input circuit

1 Opto coupler circuit

This circuit is used to detect mains voltage via a capacitor and a resistor forming a potential divider so that a lower voltage can be used to drive an opto-isolator that protects the processor reading the signal.

2 Calculations

A potential divider using a capacitor and a resistor is used to lower mains voltage to levels that can drive a typical opto-coupler input ($\approx 2V$).

A potential divider using a capacitor and resistor means using the complex identity for the capacitors reactance, X.

$$X = \frac{-j}{\omega C}$$

The ωC term is dependent on frequency and is equivalent to $2.\pi.f$.

Using a potential divider to determine the voltage over the resistor gives:

$$V_{out} = V_{in} \times \frac{R}{R - \frac{j}{2.\pi.f.C}}$$

The equation above leaves a complex divisor. To get a complex number as the numerator, the denominator and numerator must be multiplied by the conjugate of the denominator, thus:

$$\frac{R}{R - \frac{j}{2.\pi.f.C}} \equiv \frac{R \times \left(R + \frac{j}{2.\pi.f.C}\right)}{\left(R - \frac{j}{2.\pi.f.C}\right) \times \left(R + \frac{j}{2.\pi.f.C}\right)}$$

This leaves a real number as the denominator, i.e. $R^2 + \frac{1}{2.\pi.f.C}^2$. The resulting complex number, X,

$$X = \frac{R \times \left(R + \frac{j}{2.\pi.f.C}\right)}{R^2 + \frac{1}{2.\pi.f.C}^2}$$

or,

$$X = \frac{R^2 + \left(R\frac{j}{2.\pi.f.C}\right)}{R^2 + \frac{1}{2.\pi.f.C}^2} \tag{1}$$

can now be evaluated for phase and magnitude. Equation 1 can be generally applied to potential dividers in figure 1.

2.1 Example calculation

At 50Hz with 240 V a.c. applied, with R at 1000 Ohms and C at 47 nF

$$\frac{1000^2 + \left(1000\frac{j}{2.\pi.50.47e-9}\right)}{R^2 + \frac{1}{2.\pi.50.47e-9}^2}$$

$$\frac{1000^2 + \left(1000 \times 67726j\right)}{1000^2 + 67726^2}$$

$$\frac{1000^2 + \left(67726000j\right)}{4.5877 \times 10^9}$$

This gives a complex number

$$\frac{1000^2 + 67726000j}{4.5877 \times 10^9}$$

i.e.

$$(216 \times 10^{-6} + 14.76 \times 10^{-3}j) \, .$$

This complex number has a magnitude of 0.0147 and an argument of 89.15 degrees (which is expected as most of the reactance comes from the capacitor). So with 240 V a.c. applied (RMS) the opto would see a signal with $0.0147 * 240 = 3.54V(RMS)$

3 ploting the voltage at the opto-coupler

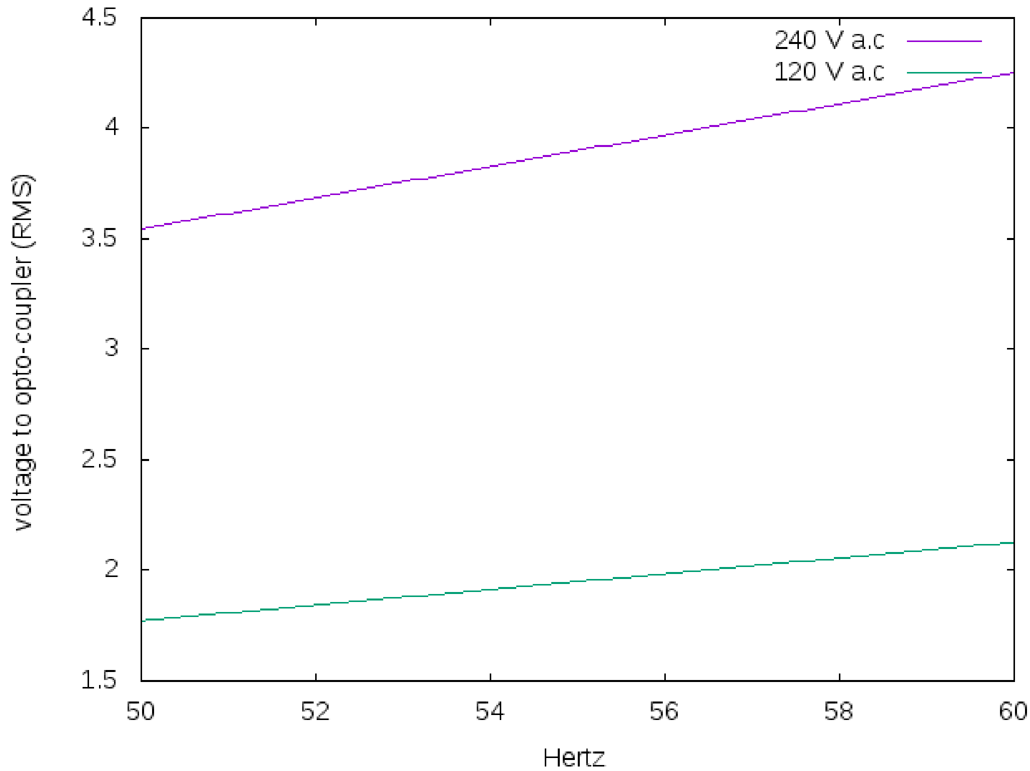

Figure 2: RMS voltage seen at opto-coupler for 50 to 60 Hz range

3.1 plotting the voltage at the opto-coupler: gnuplot scripts

```
########################################################
#
p=3.14159265358979323844
#
# 47nF
C=47e-9
#
# 1k Ohms
R=1000

# define complex operator
j={0,1}

set xlabel "Hertz"
set ylabel "Resistance"

# x is the frequency
set xrange[50:60]

# z(x) is the reactance
z(x)=(j/(2*p*x*C))

# denominator
d(x)=(R*R+z(x)*z(x))

# numerator
n(x)=(R*R+R*z(x))

plot abs(z(x)) title "reactance over capacitor"
!sleep 4

set ylabel "denominator value (abs)"
plot abs(d(x))
!sleep 4

set ylabel "numerator value (abs)"
plot abs(n(x))
!sleep 4

v(x)=abs((n(x))/(d(x)))

# gives large numbers h(x)=arg((n(x))/(d(x)))

set ylabel "voltage to opto-coupler (RMS)"
plot 240*v(x) title "240 V a.c", 120*v(x) title "120 V a.c"
!sleep 4

set terminal png
set output "RMS_volts_to_opto.png"
plot 240*v(x) title "240 V a.c", 120*v(x) title "120 V a.c"

#set angles degrees
#set label "phase change in mains over opto"
#plot 240*h(x) title "240 V a.c", 120*h(x) title "120 V a.c"
#!sleep 4
#
```

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