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OBSERVABILITY AND SENSOR POSITIONING FOR PROCESSES DESCRIBED BY LINEAR-BILINEAR EQUATIONS

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Abstract

The control of a process requires a minimum of representative information on its operating state. This information can be obtained by both direct sensors or indirectly using mathematical models. It frequently occurs that the number of direct measures is not sufficient to allow unmeasured variables to be deduced and additional sensors have to be installed to render the required variables observable. This paper presents a new technique to place sensors in systems described simultaneously by linear and bilinear equations. It takes into account placement constraints and a complete observability is obtained. Moreover it is pointed out that, from an observability point of view, a bilinear system may be studied by considering a succession of linear systems.

1. Introduction

In all the studies, where physical phenomenon occur, the problem of state estimation is analysed. Indeed, the knowledge of location and reliable information is of primary importance because it establishes good conditions for the results of every decision taken. This problem consists of approximating the true values, with estimated values which verify the model equations and are as near the measures as possible. But this estimate can only be made if the process considered is observable. Much work has already been published on the analysis of static system observability. The early work concerned the study of linear systems and are probably stemmed from [1], where [2] and [3] have largely helped to develop this analysis. The algorithms of observability which have been proposed are generally based on the graph theory [4] or on a classification of the variables, from a projection matrix [5]. After the analysis phase, which demonstrates the redundancies but also the eventual weaknesses of instrumentation system, the study of the ways of making a system observable is proposed, so that the cost engendered by the addition of sensors is as low as possible. To reach this result, one acts, in the on hand, on the locating of the sensors and, on the other hand, on their number. At that time, only a few studies on the conception of an instrumentation system which respect the constraints, such as these stated above, had been done. Studies concerning the observability and the placing of optimal sensors on electrical networks were proposed by [6]. This study shows two possible analysis: numerical and the topological. [7] has worked largely in the numerical domain. [8] has directed this study by basing itself on the topology of networks. We must also mention the work of [9], who proposes a solution based on the analysis of the incidence matrix of the graph associated to the processes, and the work of [10], who formalises the problem in terms of linear programming, that is to say, which optimisation variables can be whole, permitting the taking into account the occurrence and the position of the measurements in the optimisation. The more developed studies are probably these ones presented by [11] and [12]. Based on the analysis of the cycles of the graph associated to the processes, these studies take into account the observability of the variables and the reliability of the sensors, which measure them, to propose an architecture of optimal instrumentation according to some criterion. [13] has formalised the problem under the constraints of costs in the n-linear case. We propose a step, which emerges directly from the optimum solution. This is realised by inserting in the observability constraint, the constraints of cost and technology. This study concerns the linear and bilinear systems in a static state. These models are often used when one establishes conservation-law results. We must note that the study considered is solely based on the structure of equations, which directs the

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process.

2. Decomposition according to the observability with minimum cost - linear systems

Let a system described by its architecture, in other words by the ties between the different material supports. These connections express the transfer of matter, energy or of information between the material supports associated to the activities and are associated to the system description variables. Among these variables which characterise the system, there are those which are easily measurable, those which are measurable with difficulty and those which are not measurable. We are interested in the systems described by a form equation:

$M X^* = 0$ (1)

where the matrix $M = (m_{ij})$ with $1 \le i \le n$ and $1 \le j \le v$ with assuming that this system is described by n equations and v variables, the vector X (v.1) represents the variables of the system. In the case of an array of mater balance-sheet, n represents the number of nodes and v the number of streams.

For a system which is already partly instrumented we will sub-divide the vector of X variables into two distinct sub-vectors [14]:

$X = [X_I \ X_{II}]^T$ (2)

with: X_I : vector of the measured variables,

X_{II} : vector of the unmeasured variables.

We are now confronted with the following problem: is the knowledge of the measure X_I sufficient to render the remaining variables of the system observable ? Otherwise, is it possible to obtain this observability by positioning other sensors ?

The incidence matrix of the network can then also be sub-divided according to the partition (2):

$M = \begin{bmatrix} M_1 & M_2 \end{bmatrix}$ (3)

where M_1 is a matrix of dimension (n.m) and M_2 is a matrix of dimension (n.(v-m)).

One can class the columns of the matrix M_2 so that the corresponding measurements which there are arranged in decreasing order of cost. The points inaccessibly to the measurement will be considered have infinite cost. Afterwards, we must rewrite the largest regular part of the matrix M_2 as the identity matrix. This transformation is realised by means of the pivot algorithm, with limitation of the pivot choice to the columns of M_2. Thus, we obtain the following decomposition:

$$M_2 = \begin{bmatrix} M_{21} & M_{22} \\ M_{23} & M_{24} \end{bmatrix} \quad (4)$$

with dimension of M_{21} = r.r, dimension of M_{22} = r.(v-m-r), dimension of M_{23} = (n-r).r, dimension of M_{24} = (n-r).(v-m-r) and where M_{21} represents the largest regular part of M with rank less or equal to n. And by separating M_1 according to the decomposition in rows of M_2, M becomes:

$$M = \begin{bmatrix} M_{11} & M_{21} & M_{22} \\ M_{12} & M_{23} & M_{24} \end{bmatrix} \quad (5)$$

with : dimension of M_{11} = r.m

: dimension of M_{12} = (n-r).m

Pre-multiplying matrix M by the regular matrix R:

$$R = \begin{bmatrix} M_{21}^{-1} & 0 \\ -M_{23}M_{21}^{-1} & I_{n-r} \end{bmatrix}$$

gives the following equivalent form:

$$RM = \begin{bmatrix} M_{21}^{-1}M_{11} & I_r & M_{21}^{-1}M_{22} \\ M_{12} - M_{23}M_{21}^{-1}M_{11} & 0 & 0 \end{bmatrix} \quad (6)$$

because $M_{24} - M_{23}M_{21}^{-1}M_{22} = 0$ (7)

Leads to the final form:

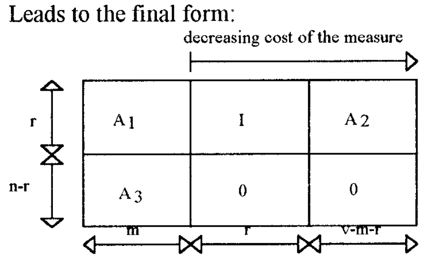

Figure 1: particular canonical form of the incidence matrix

This matrix can be read in the following way. The second row gives the redundancy equations between the variables already measured, notice that all the measured variables are not necessarily present in these equations. Finally the non-null rows of block A_2 prevent any further deductions. Thus the system is globally observable if block A_2 does not exist. Transform these variables to become observable, needs to add at least (v-m-r) measure points. They correspond to weak cost emplacements because the variables are arranged in decreasing order of their measure cost. In other words, to obtain the globally observability of the industrial process with minimal cost, we must add on the installation these (v-m-r) supplementary sensors.

3. Case of bilinear systems

We present as an example the case of systems in which we have two types of measures like a volume flow and a density or a volume flow and a temperature ... This very common case takes on particular importance in industry. Generally the bilinear systems are described by equations or the type:

$M X1 = 0$ (8)

$M (X1 .* X2) = 0$ (9)

where X1 and X2 are respectively the vectors of the variable in X1 and X2 and have the same dimensions.

The product .* corresponds to the term by term product of two vectors. We consider a process for which the structure and the equipment have been partly defined. The variables linked with this installation can be classified according to their accessibility to the measurement. Thus several categories of variables could have been worked out: the variables measurable in X1 and X2, the variables only measurable with X1, those only measurable with X2 and the variables being unmeasurable. Some economic or technical reasons may be at the origin of this partition. The problem can be posed as follows: what is the minimal number of sensors and what is their location in order to get an overall observability of the installation at an optimal cost ? Let's consider an elementary system (the M matrix is then reduced to a row). If we call n1 the number of the unmeasured variables in X1 and n2 the number of the unmeasured variables in X2 and knowing that the number of variables is equal to v (n1 and n2 $\leq$ v), the conditions of observability of an elementary bilinear system can be listed one by one [14] or be represented by a two level-tree (or shaft) [14], and they can be written as follows:

a) $n1 \leq 2$ and $n2 \leq 1$ (10)

b) $n1 + n2 \leq 2$ (11)

For a non elementary system (that is to say that the M matrix possesses at least two rows), the study of the bilinear systems observability will be performed according to a hierarchic diagram. In a first stage we complete the list of the observable variables in X1 (OBSX1 list) from putting the matrix M in a particular canonical form beside the X1 magnitude. Then at a second stage we do the same for the X2 magnitude, of which we take each equation (row) and we check for n2 = 0 and n2 = 1 if the relationship (12) proves to be true. If it does so, we complete the list of the observable variables in X2 (OBSX2 list) and the list of the observable variables in X1.

$$n1' + n2 \leq 2 \quad (12)$$

where n2 is the number of unmeasured variables in X2 in equation (9) and n1' is the number of non observable variables for this equation in X1 for the same equation (not belonging to the OBSX1 list). The algorithm to treat the observability of bilinear systems is hierarchical. It uses the particular canonical form of the matrix M defined above. During this study, we have to detect the equations for which the number of unmeasured variables respect the observability constraints of bilinear system. At first, we test the stability of the lists which contain the observability variables in X1 and X2. When the contents of these lists is not modified, we stop the treatment. The algorithm of minimisation of the instrumentation cost of the bilinear systems is then written as:

Stage 1: Looking for the particular canonical forms M1 and M2 that correspond to X1 and X2 magnitudes in the same way as in the linear system.

Stage 2: Making in M2 the measure of all the variables corresponding to the columns that do not contain a pivot so that the maximum number of unmeasured variables per equation in X2 is n2 = 0 (redundancy equation) or n2 = 1 (deduction equation).

Stage 3: Making in M1 the measure of all the variables for which the columns do not contain any pivot and for which we have more than two missing measurement per equation. In this case the maximum number of measured variables in X1 is either n1 = 2, n1 = 1 (deduction equation) or n1 = 0 (redundancy equation).

Stage 4: Launching the algorithm calculating OBSX1 and OBSX2. If OBSX1 and OBSX2 are complete, that is to say that all the variables are observable, we stop. Then we get the observability at an optimal cost by measuring the variables that need to be measured described in the second and third stages. Otherwise we carry out the following stage.

Stage 5: We measure the least expensive variable in M1 or in M2. Various cases appear:

- if OBSX1 and OBSX2 are complete, we stop.
- if this variable measure does not modify the lists OBSX1 and OBSX2 we cancel its measurement.
- if OBSX1 and OBSX2 are not complete we do the stage 5 again until we obtain in M1 only one variable of which the column does not contain any pivot.

Stage 6: When only one variable of which the column does not contain any pivot remains in M1, two cases can appear:

- if the variable to be measured (minimum cost) is in M1, we measure it (OBSX1 and OBSX2 will be complete).
- if the variable(s) to be measured are in M2, we measure these variables as long as OBSX1 and OBSX2 are not complete but at the same time as soon as their measurement costs exceed the one of the M1 variable of which the column does not contain any pivot, we cancel all these measures to measure M1, OBSX1 and OBSX2 then will be complete.

4. Application

Let's consider a schematised process in a network form Figure 2 made up of 3 nodes and 7 streams.

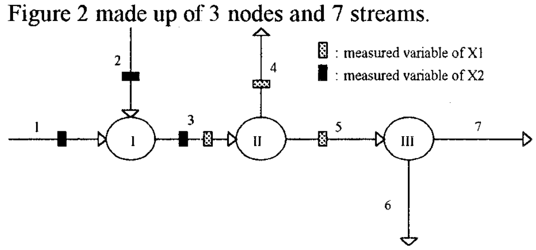

Figure 2: 3 nodes and 7 streams network

An instrumentation architecture of this process is given by the table 1 for the X1 magnitude andfor the X2 magnitude.

unmeasured variables on	Sensor weights	unmeasured variables on	Sensor weights
X1		X2	
1	15	4	10
2	20	5	30
6	5	6	20
7	25	7	*

Table 1: Costs associated with the measures in X1 and the measures in X2

From the tables 1, we will study the configurations which permit to obtain the observability of the array with the least cost. First we observe the different combinations which permit us to respect the observability constraints. The following table 2 summarises these combinations.

combi nation s	measurements of the variables of X1	measurements of the variables of X2	measure ment costs
1	6	4 et 6 or 5 et 6	35 or 55
2	7	" "	55 or 75
3	1 and 6	" "	50 or 70
4	1 and 7	" "	70 or 90
5	2 and 6	" "	55 or 75
6	2 and 7	" "	75 or 95
7	6 and 7	4, 5 and 6	90
8	1 and 6	" "	80
9	2 and 6	" "	85
10	1 and 7	" "	100
11	2 and 7	" "	105

Table 2: Measurement combinations costs

Afterwards, we take the combination with the minimum cost. The combination, which has for function cost equal to 35, gives the observability with minimum cost by doing the measure of the variable 6 at X1 and the variables 4 and 6 at X2. Let's apply the algorithm of the cost minimisation to this bilinear system. The matrix of M corresponding to the X2 magnitude is written:

	Variable	1	2	3	7*	5_3	6_{20}	4_{10}
	Equation							
	I	1	1	-1	0	0	0	0
M =	II	0	0	1	0	-1	0	-1
	III	0	0	0	-1	1	-1	0

The first stage applied to the M matrix gives its particular canonical form M2:

	Variable	1	2	3	7*	5_{30}	6_{20}	4_{10}
	Equation							
	III'	0	0	-1	1	0	1	1
M2 =	II'	0	0	-1	0	1	0	1
	I'	1	1	-1	0	0	0	0

The matrix of M incidence corresponding to the X1 magnitude is written:

	Variable	3	4	5	7_{25}	2_{20}	1_{15}	6_5
	Equation							
	I	-1	0	0	0	1	1	0
M =	II	1	-1	-1	0	0	0	0
	III	0	0	1	-1	0	0	-1

M in its particular canonical form is written:

	Variable	3	4	5	7_2	2_2	1_{15}	6_5
	Equation							
	III'	0	0	-1	1	0	0	1
M1 =	I'	-1	0	0	0	1	1	0
	II'	1	-1	-1	0	0	0	0

The second stage gives the following result: in order that the number n2 of the variables not being measured by the equation in X2 is smaller than 1 or equal to 1, in M2, we measure the variables 6 and 4. Their measurement costs are worth (20 + 10) = 30, therefore M2 becomes MII.

	Variable	1	2	3	4_{10}	6_{20}	7*	5_{30}
	Equation							
	III'	0	0	-1	1	1	1	0
MII =	II'	0	0	-1	1	0	0	1
	I'	1	1	-1	0	0	0	0

The third stage does not generate any alteration. Indeed the number n1 of the unmeasured variables by equation in X1 is smaller than two or equal to two, so we do not make a measurement on the X1 in magnitude in M1. The 4th stage allow the calculations of the OBSX1 and OBSX2 lists (to simplify the writing we only make a note in those lists of the observable variables index) and we obtain: OBSX1 = (3, 4, 5).

In order to calculate OBSX2, in MII we repeat the equations for which all is calculated in X2 (equation I') and we check if the number n1' of the non observable variables corresponding in X1 is smaller than 2 or equal to 2. If it is so, we complete the OBSX1 and OBSX2 lists with these variables. In our case, this is expressed by:

- the analysis of the MII matrix equation I' that gives: OBSX1 = (3, 4, 5, 1, 2), OBSX2 = (1, 2, 3).

In MII we spot the equations for which a variable is unmeasured in X2 (equations II' and III') and we check if the number n1' of non observable variables in X1 for these equations is smaller than 1 or equal to 1. If it is so, we complete the OBSX1 and OBSX2 lists.

- Then the analysis of the equation III' does not allow to complete OBSX1 = (3, 4, 5, 1, 2) and OBSX2 = (1, 2, 3) because the number of non observable variables in X1 for this equation is higher than 1.

- The analysis of the equation II' leads to the result: OBSX1 = (3, 4, 5, 1, 2) and OBSX2 = (1, 2, 3, 4, 5). We notice that OBSX1 and OBSX2 are not complete. Then the stage 5 is undertaken.

We are going to measure the least expensive variable in MI or in MII, in our case we are going to measure the variable 6 in MI, its cost of measure is 5. In its particular canonical form MI then becomes MI:

MI =

Variable	3	4	5	6_5	7_{25}	2_2	1_{15}
Equation							
III'	0	0	-1	1	1	0	0
I'	-1	0	0	0	0	1	1
II'	1	-1	-1	0	0	0	0

from MI and MII we complete OBSX1 and OBSX2 again. We obtain: OBSX1 = (3, 4, 5, 6, 7). The analysis of the MII equations is made up of:

- the analysis of the I' equation that allows us to complete OBSX1 = (3, 4, 5, 6, 7, 1, 2) and OBSX2 = (1, 2, 3).
- the analysis of the III' equation that gives: OBSX1 = (3, 4, 5, 6, 7, 1, 2) and OBSX2 = (1, 2, 3, 4, 6, 7).
- the analysis of the II' equation that gives the final result: OBSX1 = (3, 4, 5, 6, 7, 1, 2) and OBSX2 = (1, 2, 3, 4, 6, 7, 5).

We note that OBSX1 and OBSX2 are complete, that is to say that all the variables are observable in X1 and X2 at minimal cost. This minimal observability cost is worth (20 +10 +5) = 35. It is made up of: 20 for the measure cost of the variable 6 in X2, 10 for the measure cost of the variable 4 in X2 and 5 for the measure cost of the variable 6 in X1. Note that the proposed algorithm gives the same observability cost as that obtained by testing all the possible observability combinations (table 2).

6. Conclusion

This paper deals with the observability of a process described by a set of linear-bilinear algebraic equations with partial measurements. From this analysis, we found a classification of the variables and the equations of the process, for the unobservable subset we define the location of new sensors to increase the observability index. The proposed strategy has been found to be a very efficient way of establishing the number and location of additional sensors. Two situations can be dealt with: the first one is concerned with an already partially instrumented process which is not yet observable, the second one concerns processes with no instruments at all.

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